Transportation, Transshipment, and Assignment Problems

Chapter 6

Chapter Topics

- The Transportation Model
- Computer Solution of a Transportation Problem
- The Transshipment Model
- The Assignment Model
- Computer Solution of an Assignment Problem
Overview

- Part of a class of LP problems known as *network flow models*.
- Special mathematical features that permit very efficient, unique solution methods (variations of traditional simplex procedure).
- Detailed description of methods is contained on the companion website.
- Text focuses on model formulation and solution with Excel and QM for windows.
- Web site Module B addresses transportation and assignment solution methods.

The Transportation Model: Characteristics

- A product is transported from a number of sources to a number of destinations at the *minimum possible cost*.
- Each *source is able to supply a fixed number* of units of the product, and each *destination has a fixed demand* for the product.
- The linear programming model has *constraints for supply* at each source and *demand* at each destination.
- All constraints are equalities in a balanced transportation model where supply equals demand.
- Constraints contain inequalities in unbalanced models where supply does not equal demand.
Transportation Model Example

**Problem Definition and Data**

How many tons of wheat to transport from each grain elevator to each mill on a monthly basis in order to minimize the total cost of transportation?

<table>
<thead>
<tr>
<th>Grain Elevator</th>
<th>Supply</th>
<th>Mill</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Kansas City</td>
<td>150</td>
<td>A. Chicago</td>
<td>220</td>
</tr>
<tr>
<td>2. Omaha</td>
<td>175</td>
<td>B. St. Louis</td>
<td>100</td>
</tr>
<tr>
<td>3. Des Moines</td>
<td>275</td>
<td>C. Cincinnati</td>
<td>300</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>600 tons</strong></td>
<td><strong>Total</strong></td>
<td><strong>600 tons</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grain Elevator</th>
<th>A. Chicago</th>
<th>B. St. Louis</th>
<th>C. Cincinnati</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Kansas City</td>
<td>$6</td>
<td>$8</td>
<td>$10</td>
</tr>
<tr>
<td>2. Omaha</td>
<td>7</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>3. Des Moines</td>
<td>4</td>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

*Transport Cost from Grain Elevator to Mill ($/ton)*

Figure 6.1  Network of transportation routes for wheat shipments
Transportation Model Example

Model Formulation

Minimize \( Z = 6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C} \)

subject to:

\[ \begin{align*}
    x_{1A} + x_{1B} + x_{1C} &= 150 \\
    x_{2A} + x_{2B} + x_{2C} &= 175 \\
    x_{3A} + x_{3B} + x_{3C} &= 275 \\
    x_{1A} + x_{2A} + x_{3A} &= 200 \\
    x_{1B} + x_{2B} + x_{3B} &= 100 \\
    x_{1C} + x_{2C} + x_{3C} &= 300 \\
    x_{ij} &\geq 0
\end{align*} \]

\( x_{ij} \) = tons of wheat from each grain elevator, \( i, i = 1, 2, 3 \), to each mill \( j, j = A, B, C \)

Transportation Model Example

Computer Solution with Excel (1 of 4)

Objective function

\[ =D5+D6+D7 \]

Decision variables in cells C5:E7

\[ =C7+D7+E7 \]

Cost array in cells K5:M7

Exhibit 6.1
Transportation Model Example
Computer Solution with Excel (2 of 4)

Exhibit 6.2

Supply constraints

Demand constraints

Transportation Model Example
Computer Solution with Excel (3 of 4)

Exhibit 6.3
Figure 6.2   Transportation network solution for wheat-shipping example

Exhibit 6.4
Transportation Model Example
Computer Solution with Excel QM (2 of 3)

1. Click on “Add Ins,” then “Excel QM” or “Taylor” to access the macro menu

2. Enter data values for problems; initially this array is blank

3. Click on “Data,” “Solver,” and then “Solve.”

Transportation Model Example
Computer Solution with Excel QM (3 of 3)

Click on “Data” tab and then “Solver”
Transportation Model Example
Computer Solution with QM for Windows (1 of 4)

Use any starting method

Exhibit 6.7

Use any starting method

Transportation Model Example
Computer Solution with QM for Windows (2 of 4)

Exhibit 6.8
Transportation Model Example
Computer Solution with QM for Windows (3 of 4)

Exhibit 6.9

Transportation Model Example
Computer Solution with QM for Windows (4 of 4)

Exhibit 6.10
The Transshipment Model

Characteristics

- Extension of the transportation model.
- Intermediate transshipment points are added between the sources and destinations.
- Items may be transported from:
  - Sources through transshipment points to destinations
  - One source to another
  - One transshipment point to another
  - One destination to another
  - Directly from sources to destinations
  - Some combination of these

Transshipment Model Example

Problem Definition and Data

Extension of the transportation model in which intermediate transshipment points are added between sources and destinations.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nebraska</td>
<td>$16</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>2. Colorado</td>
<td>15</td>
<td>14</td>
<td>17</td>
</tr>
</tbody>
</table>
Transshipment Model Example
Transshipment Network Routes

Minimize $Z = 16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24}$
+ $17x_{25} + 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47}$
+ $11x_{48} + 4x_{56} + 5x_{57} + 12x_{58}$

subject to:

$x_{13} + x_{14} + x_{15} = 300$
$x_{23} + x_{24} + x_{25} = 300$
$x_{36} + x_{46} + x_{56} = 200$
$x_{37} + x_{47} + x_{57} = 100$
$x_{38} + x_{48} + x_{58} = 300$

$x_{13} + x_{23} - x_{36} - x_{37} - x_{38} = 0$
$x_{14} + x_{24} - x_{46} - x_{47} - x_{48} = 0$
$x_{15} + x_{25} - x_{56} - x_{57} - x_{58} = 0$

$x_{ij} \geq 0$

Supply constraints for farms in Nebraska and Colorado
Demand constraints at the Chicago, St. Louis and Cincinnati mills

Figure 6.3 Network of transshipment routes
Exhibit 6.11

Objective function:

\[ \text{Objective function} = \text{SUM(B6:B7)} \]

Cost arrays:

\[ \text{Cost arrays} = \text{SUM(B6:D6)} \]

\[ \text{Cost arrays} = \text{SUM(C13:C15)} \]

\[ \text{Cost arrays} = \text{SUM(C13:E13)} \]

Constraints for transshipment flows; i.e., shipments in = shipments out

Exhibit 6.12

Transshipment constraints in cells C20:C22
The Assignment Model Characteristics

- Special form of linear programming model similar to the transportation model.
- **Supply** at each source **and demand** at each destination **limited to one unit**.
- In a balanced model supply equals demand.
- In an unbalanced model supply does not equal demand.
Assignment Model Example

Problem Definition and Data

Problem: Assign four teams of officials to four games in a way that will minimize total distance traveled by the officials. Supply is always one team of officials, demand is for only one team of officials at each game.

<table>
<thead>
<tr>
<th>Game Sites</th>
<th>Raleigh</th>
<th>Atlanta</th>
<th>Durham</th>
<th>Clemson</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>210</td>
<td>90</td>
<td>180</td>
<td>160</td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>70</td>
<td>130</td>
<td>200</td>
</tr>
<tr>
<td>C</td>
<td>175</td>
<td>105</td>
<td>140</td>
<td>170</td>
</tr>
<tr>
<td>D</td>
<td>80</td>
<td>65</td>
<td>105</td>
<td>120</td>
</tr>
</tbody>
</table>

Minimize \( Z = 210x_{AR} + 90x_{AA} + 180x_{AD} + 160x_{AC} + 100x_{BR} + 70x_{BA} 
+ 130x_{BD} + 200x_{BC} + 175x_{CR} + 105x_{CA} + 140x_{CD} 
+ 170x_{CC} + 80x_{DR} + 65x_{DA} + 105x_{DD} + 120x_{DC} \)

subject to:
\[
\begin{align*}
x_{AR} + x_{AA} + x_{AD} + x_{AC} &= 1 \\
x_{BR} + x_{BA} + x_{BD} + x_{BC} &= 1 \\
x_{CR} + x_{CA} + x_{CD} + x_{CC} &= 1 \\
x_{DR} + x_{DA} + x_{DD} + x_{DC} &= 1 \\
x_{AR} + x_{BR} + x_{CR} + x_{DR} &= 1 \\
x_{AA} + x_{BA} + x_{CA} + x_{DA} &= 1 \\
x_{AD} + x_{BD} + x_{CD} + x_{DD} &= 1 \\
x_{AC} + x_{BC} + x_{CC} + x_{DC} &= 1 \\
x_{ij} &\geq 0
\end{align*}
\]
Assignment Model Example
Computer Solution with Excel (1 of 3)

Objective function

Decision variables, C5:F8

Objective function: =C5+D5+E5+F5

Mileage array

Mileage array: =D5+D6+D7+D8

Exhibit 6.13

Assignment Model Example
Computer Solution with Excel (2 of 3)

Simplex LP

Simplex LP

Exhibit 6.14
Assignment Model Example
Computer Solution with Excel (3 of 3)

Exhibit 6.15

Assignment Model Example
Assignment Network Solution

Figure 6.5 Assignment network solution for ACC officials
Assignment Model Example
Computer Solution with Excel QM

Exhibit 6.16

Assignment Model Example
Computer Solution with QM for Windows (1 of 2)

Exhibit 6.17
A concrete company transports concrete from three plants to three construction sites. The supply capacities of the three plants, the demand requirements at the three sites, and the transportation costs per ton are as follows:

<table>
<thead>
<tr>
<th>Construction site</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Supply (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>$8</td>
<td>$5</td>
<td>$6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>10</td>
<td>12</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>9</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>Demand (tons)</td>
<td>150</td>
<td>70</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Determine the linear programming model formulation and solve using Excel.
Example Problem Solution
Model Formulation

Minimize \( Z = 8x_{1A} + 5x_{1B} + 6x_{1C} + 15x_{2A} + 10x_{2B} + 12x_{2C} + 3x_{3A} + 9x_{3B} + 10x_{3C} \)

subject to:

\[
\begin{align*}
  x_{1A} + x_{1B} + x_{1C} &= 120 \\
  x_{2A} + x_{2B} + x_{2C} &= 80 \\
  x_{3A} + x_{3B} + x_{3C} &= 80 \\
  x_{1A} + x_{2A} + x_{3A} &\leq 150 \\
  x_{1B} + x_{2B} + x_{3B} &\leq 70 \\
  x_{1C} + x_{2C} + x_{3C} &\leq 100 \\
  x_{ij} &\geq 0
\end{align*}
\]

Example Problem Solution
Computer Solution with Excel

[Excel spreadsheet showing the example problem solution]
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